Numerical Prediction of Rhombic Rotational Magnetization Patterns in a Transformer Core Package

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Abstract – Evaluations of local induction time patterns \( B(t) \) in transformer cores show high relevance for both losses and magnetostriction. This paper presents numerical calculations for a 3-phase core package stacked from GO SiFe for \( B_{SOM} = 1.7 \) T. Modelling is based on a novel Multi-directionally Nonlinear Magnetic Equivalence Circuit Calculation (MACC). It considers non-linear permeability functions in rolling direction (RD), transverse direction (TD), and diagonal direction (DD) in overlaps. MACC yields instantaneous local values \( B \), and the corresponding reluctances and permeabilities as a basis for conclusions. Snapshots of induction distributions for important time instants of zero or maximum limb induction reveal dominant roles of anisotropy and multi-directional non-linearity. Small changes of permeability in TD yield distinct changes of rotational magnetization (RM) and circulating magnetization (CM). Local dynamic magnetization patterns \( B(t) \) are calculated considering 180 instants of time, for sufficient resolution of dynamics. The results confirm the formation of RM patterns of oblique rhombic (or lozenge) shape, in contrast to elliptic patterns as frequently assumed. They also confirm that the induction vector \( B \) rotates with maximum angular velocity when passing through the transverse direction.

Index Terms — Numerical magnetic modelling, electric equivalence circuits, transformer cores, induction distribution, rotational magnetization, circulating magnetization, non-linear systems.

I. INTRODUCTION

It is well known, that both core losses and magnetostriction of transformer cores depend significantly on the local induction values \( B \). As a very rough rule, a 1% increase of \( B \) causes 2% increase of the losses [1]. It means that even very small variations of \( B \) may have high impact on the magnetic core performance. Furthermore, losses and magnetostriction increase distinctly under rotational magnetization (RM), as being typical for T-joints of 3-phase cores. Due to RM, the losses in T-joints may increase up to 2 times while the magnetostriction up to 10 times [2]. This means that effective evaluations of local induction values - for both rolling direction (RD) and transverse direction (TD) - are of a high relevance for the core manufacturer in order to optimize designs and materials with respect to losses and magnetostriction-caused noise.

The conventional tools for local induction measurements are search coils placed in holes, drilled through core laminations [2-
Such measurements are extremely laborious, apart from causing impairments to the magnetic core. Furthermore, many search coils are needed for a good spatial (geometrical) resolution. Some novel methods like pin sensor [6] and tangential induction sensor [7] can be applied with reduced expenditure for evaluations in the entire core with very high resolution. However, all measurement methods can be performed only in a laboratory with direct access to a given core.

Since the measurement-methods are time consuming and laborious, many authors used numerical methods like FEM for the estimation of local induction values. However, to avoid unacceptably high expenditure, they tend to consider the very important role of non-linearity in restricted ways. For example, the flux distribution of a 3-limb, 3-phase transformer core is presented in [8], but permeabilities in rolling direction (RD) and transverse direction (TD) are assumed constant as \( \mu_{RD} = 3500 \) and \( \mu_{TD} = 350 \). In [9], a Gaussian equation is used for the variation of the induction and permeability in RD and in the joints, but for the TD a constant value \( \mu_{TD} = 10000 \) is used. As an exception, [10] considers non-linear permeabilities \( \mu_{RD} \) and \( \mu_{TD} \), but restricts modelling to the transformers core joints. As a main problem of FEM-methods, high computational times are needed even for a single instant of time, time transient calculations resulting as a challenge [11].

An alternative to FEM is given by equivalent circuit modelling (ECM). The review [11] ends with the following statement: “Results in the literature suggest that the magnetic equivalence circuit approach has more promise than finite element analysis as a design tool”. In the case of transformers, ECM is mainly used for global estimations of the machines performance in very rough ways, including the windings and magnetic core. These models are based on the circuit theory and the duality principle. Both the electrical part of the transformer (source, primary and secondary windings, harmonics, etc.) and the equivalent magnetic part (equivalent magnetic resistances for the rolling direction of limbs and yokes) are combined into an electrical circuit. However, ECM tends to be restricted to very few elements, corresponding to entire sections of limbs and yokes (e.g. [12,13]). To our knowledge, a complete ECM that considers the transverse direction – or even the normal direction of a core - has not been presented so far.

The present paper discusses a novel approach for the evaluation of local induction distributions. The so called Multi-directionally Non-linear Magnetic Equivalence Circuit Calculation (MACC) was presented for the first time in [14, 15]. The basic advantage of this model is a rapid and flexible simulation of flux distributions with consideration of multi-directional non-linearities of the core material. Furthermore, the output of the model is not given by flux lines, but by numerical local, instantaneous values of all three induction, reluctance and permeability. This makes the model transparent and presentative, and it favours quantitative conclusions on losses or magnetostriction. Flux elements can be modified in simple, flexible ways, thus supporting physical interpretations through variations of core geometry or/and material parameters.

The reference [14] describes 3D flux distributions in a simple arrangement of a soft magnetic ribbon with a short further ribbon on top in asymmetric position. The reference [15] describes fundamental aspects of MACC at the example of 2D flux distributions of a 3-phase transformer core package, for the
time instant of zero-flux in the T-limb. The aim of the present paper is to demonstrate possibilities of dynamic modelling as well as possibilities for versatile interpretations of results. The latter comprise snapshots of instantaneous induction distributions as well as local dynamic patterns $B(t)$ of the induction vector with focus on rotational magnetization. For the first time, instantaneous induction values are complemented by instantaneous data also of reluctances and permeability values, as a significant support for physical conclusions.

II. NUMERICAL MACC-MODELLING OF TRANSFORMER CORE

The model was tested on a 3-phase, 3-limb transformer core package (Fig.1) stacked from grain oriented material C130-30 with the following dimensions: width 120 cm, height 100 cm, stacking height 10 cm and lamination width 20 cm. Furthermore, we assume a multistep-lap construction of five steps in the overlaps of corners and T-joint.

![Fig.1 The considered 3-phase core package, after [15].](image)

The model considers two main flux paths, an inner one and a peripheral one. Each path is simulated by a series of magnetic reluctances in RD. The interaction between the two paths is given by reluctances in TD. In quite easy ways, it would be possible to consider also other angles $\psi$ in the plane ($\psi$ is the angle between $B$ and the RD). However, it should be reminded to their links to crystallographic main axes. Of course, the RD represents the easiest direction of magnetization due to its small angles to the axes [001] of the individual grains. But also the TD is supported due to being close to [010] and [100] though they are tilted. On the other hand, between the RD and TD, we find the “hard directions” ($\psi \approx 55^\circ$), as clearly characterized by weak induction values during rotational magnetization. This – as well as the effectiveness of the results of modelling - should justify that flux paths in other angles but $0^\circ$ and $90^\circ$ are not assumed. As an exception, we consider the "diagonal" direction DD of overlap regions as resulting from two RDs orthogonal to each other. For the simulations, an overall of 47 regional elements are used, 27 reluctances in RD, 14 ones in TD and six ones in DD.

In each sub-region, the reluctances in RD and in TD, respectively, are calculated according to:

$$R_{RD} = \frac{L}{(A \mu_o \mu_{RD})} = \frac{L}{(WH \mu_o \mu_{RD})}, \quad (1)$$

$$R_{TD} = \frac{L}{(A \mu_o \mu_{TD})} = \frac{L}{(WH \mu_o \mu_{TD})}. \quad (2)$$

Here, $A$ is the flux path cross section, $L$ the effective path length, $W$ the width and $H$ the height of the corresponding element, $\mu_o$ the permeability of free space. $\mu_{RD}$ and $\mu_{TD}$ are the relative permeability values in RD and TD, respectively.
permeabilities in RD and TD, respectively. $\mu_{RD}$ and $\mu_{TD}$ are obtained from specific measurements or from catalogue values of the investigated type of material considering the non-linearity in both directions (compare Fig.2a,b). For the investigated material C130-30, the highest permeability in RD ($\mu_{RD,\text{MAX}} \approx 35000$) is given for $B \approx 1.2$ T, corresponding to low reluctance. The highest permeability for TD ($\mu_{TD,\text{MAX}} \approx 4500$) is estimated for $B \approx 1$ T.

![Graphs](image)

**Fig. 2.** Data for the calculation of regional reluctances. (a) The permeability $\mu_{RD}$ in RD as a non-linear function of the induction $B_{RD}$ [16]. (b) The permeability $\mu_{TD}$ in TD as a non-linear function of the induction $B_{TD}$ (estimated from [17]). (c) The overlap reluctance as a non-linear function of the induction $B_{DD}$ for multistep-lap construction with five steps (estimated after [18]).

The values of the reluctances in the diagonal directions $R_{DD}$ of the overlap regions of corner and T-joint area are estimated from data taken from previous experiments [18] for the case of multi-step-lap construction with $N = 5$. Fig.2c shows that the values below 1 T are negligibly low. On the other side, strongly enhanced reluctances are given for inductions bigger than 1.2 T, according to strong non-linearity.

Using the principle of duality, an equivalent resistant circuit model is established. The flux is impressed by the three sine sources $\Phi_R, \Phi_S$ and $\Phi_T$ in R, S and T-limb, respectively, with phase shifts of 120°. The simulations were performed for $B_{\text{NOM}} = 1.7$ T. Using the MACC model [15], the local induction distributions were calculated for 180 instants of time by means of an in-house developed MACC-software established in MATLAB. As closer described in [15], MACC represents a kind of parameter harmonization method. The basic idea is to vary all considered local induction values as long until they all agree to the corresponding, adapted, local permeability values according to the three non-linear functions of Fig. 2. In principle, this could be attained through a probability/permutation procedure, which however would need very long computing times. A speed-up is attained by a simple hill climb algorithm that is closer described in [15].

For the present core, we started the approximation process assuming 1.5 T in all RD-elements, $1.5/\sqrt{2}$ T in DD-elements, and 0.1 T in TD-elements. With this starting point, 1% approximation proved to need a calculation time of about 1s for instantaneous modelling. The calculation of all local induction values for 180 instants of time needs about three minutes.

It should be noticed that “harmonization methods” for transformer cores were reported also elsewhere (e.g. [19]). However, other targets were given, in connection to other
methodologies that differ from the here applied in fundamental ways.

III. LOCAL INDUCTION DISTRIBUTIONS FOR INSTANTS OF MAXIMUM LIMB FLUX

As already mentioned, the calculations were performed for 180 time instants as being significant for sufficient resolution of dynamic features as given in Fig. 8 of Section V. However, for a better overview of the involved flux distributions and for a deeper understanding of the complex formation mechanisms of rotational magnetization (RM) and circulating magnetization (CM), we start with some instantaneous snapshots. In the following, they concern the cases of maximum induction in the middle and the outer limb, respectively.

Fig. 3 shows results for the time instant of maximum induction in the S-limb, corresponding to the nominal value $B_{\text{nom}} = 1.7$ T. The arrows in Fig. 3b indicate local directions of induction paths in RD, TD and DD, respectively. The bold print numbers give the intensities of induction $B$ in T or mT, respectively. In Fig. 3a, the numbers in italics give the reluctance values $R$ of the corresponding flux elements in kA/Vs = k H$^{-1}$. The reluctances offer an effective assessment of effects of non-linearity. However, possibilities of quantitative comparisons between single reluctances are restricted by the fact that length $L$ and cross section $A$ according to equations (1) and (2) are not constant for all elements. Thus, in the following, the corresponding comparisons are restricted to elements of induction of identical $L$ and $A$. On the other hand, direct comparisons are offered by the local permeability values $\mu$ as given in Fig. 3c in underlined values.

The given instant (Fig. 3b) is represented by a symmetric split of flux, according to mean values of 0.85 T in both outer limbs. For a closer discussion, let us consider the right half of the core. Since the calculation is restricted to two flux paths, the model yields homogeneous magnetization in the S-limb. Passing through the overlap into the T-joint, most of the flux remains in the inner path due to the very high anisotropy of the material. In fact, the majority of flux "travels" like on rails around the window with high intensity even in the center of the T-limb.

Within the T-joint, just 10% of flux enter the yokes peripheral path in the course of TD-flux. The latter is distributed all along the yoke and the T-limb. The corresponding TD-induction values decrease strongly. This tendency is supported by the strong non-linearity of TD (Fig. 2b), the TD-permeability values $\mu_{\text{TD}}$ decreasing from about 2000 (1.9k) in the T-joint down to the initial permeability value of 500 in the middle of the T-limb.
Fig. 3. Local results of instantaneous modelling for the moment of maximal induction in the middle S-limb. The width of arrows or bars offers a rough coding of intensities. (a) The equivalent magnetic circuit with reluctances $R$ in k H$^{-1}$. (b) Induction values $B$ in T or mT. (c) Relative permeabilities $\mu$.

Apart from the high anisotropy and the non-linearity of $\mu_{TD}$, the flux concentration round the window of course is also due to the fact that the inner path length is shorter than the peripheral one. However, this mechanism is weakened through the very strong non-linearity of $R_{DD}$ (Fig. 2c). At the window side, the latter yields the high reluctance value of 0.42 k H$^{-1}$, in contrast to only 0.01 k H$^{-1}$ at the periphery. On the other hand, indirectly the peripheral path is elongated by the non-linearity of $\mu_{RD}$, all along this path, the reluctances being higher than along the window.

Summarizing, the given instant is characterized by a high degree of inhomogeneity. It results from a complex superposition of high anisotropy and non-linearities in the considered three directions. The instantaneous inhomogeneity can be assumed to contribute to a general distortion of flux that results in increased eddy current losses and enhanced higher harmonics of magnetostriction.

Fig. 4. Local results of instantaneous modelling for the moment of maximal induction in the outer R-limb. In bold, the induction values $B$ in T or mT. In italics, reluctances $R$ in k H$^{-1}$. Underlined, relative permeabilities $\mu$.

Fig. 4 depicts results for the instant of maximal induction in an outer limb. The flux is excited in the R-limb and is taken up
in two equal parts by the middle S-limb and the outer T-limb. The flux distribution in the R-limb is quite homogeneous. However, a small part of the peripheral path flux enters through the transverse reluctances into the inner path.

The left part of the yoke shows a slight flux de-load of the inner path. In contrast to the R-limb where a weak TD-flux flows towards the window, in the left yoke part, a small flux portion passes through the TD back into the periphery, “trying” to avoid the high overlap reluctance ($R_{DD} \approx 3.3 \text{ kH}^{-1}$) of the T-joint. As a result of the strong non-linearity of $\mu_{TD}$, this TD-flux shows strong concentration in the T-joint. In the latter, almost the entire flux from the inner path of the left yoke half passes into the left part of the middle limb. It causes an overload of the left side and a strong inhomogeneous flux distribution. This is also supported by the non-linearity of $\mu_{RD}$ (see Fig.2a). At the left, the latter is close to 30k while just 20k at the right.

After passing the T-joint, a peripheral flux de-load arises all along the right yoke half and the outer T-limb. The corresponding TD-induction values decrease in an exponential way which again can be interpreted by the strong non-linearity of $\mu_{TD}$. It decreases from 2500 in the T-joint down to 400 in the central part of T-limb.

Summarizing, this time instant is characterized by a homogeneous flux distribution in the R-limb and in the left yoke, while the distribution in the middle limb and the right yoke tends to be strongly inhomogeneous. The balancing role of the overlap reluctances of the right corner is not significant for the given instant of time.

**IV. LOCAL INDUCTION DISTRIBUTIONS FOR INSTANTS OF ZERO LIMB FLUX**

As for example discussed in [20], the case of zero limb-flux of a 3-phase core is linked with distinct rotational magnetization (RM) in the T-joints but also in adjoining regions. In this section, an interpretation of the complex formation of RM is presented, based on the local induction values, the corresponding reluctances in RD, TD, DD and the relative permeability values.

Fig.5a depicts the time instant when the magnetic flux with induction $B = 1.7 \text{ T cos } 30^\circ = 1.47 \text{ T}$ is excited in the R-limb and finally taken over by the T-limb. The total flux, passing through the middle S-limb is zero. While maximum flux in the S-limb is linked with vertical symmetry of the flux distribution, this case shows anti-symmetry.
Fig. 5. Instantaneous results for zero-flux in a limb. In bold, local induction values $B$ in T or mT. In italics local reluctances $R$ in k H$^{-1}$ in RD, TD and DD. Underlined: local instantaneous permeabilities $\mu$.

(a) For the instant of zero induction in the middle S-limb. (b) For the instant of zero induction in the outer T-limb.

Fig. 5a shows that roughly half of the total flux flows along the periphery with small variations of intensity. A distinct increase up to 1.66 T is restricted to the T-joint region due to some flux uptake from the inner path. Inversal tendencies are given for the inner path where an induction decrease arises through the T-joints V-segment of high magnetic resistance. Some portion of the inner flux flows into the periphery trying to avoid the V-segment. The corresponding balancing fluxes in TD increase gradually, from 10 mT next to the left corner up to 93 mT shortly before the T-joint. Again, this concentration is due to non-linear $\mu_{TD}$ (Fig. 2b). It leads to a strong inhomogeneity in the T-joint, somehow restricted by the high anisotropy of the material.

In the T-joint region, a significant part of the flux from the inner path enters through a sharp 90° turn into the middle S-limb. There, the flux splits into significant portions:

(i) One portion passes over to the right part of the yoke through the relatively low TD-reluctance $R_{TD} = 2.1$ k H$^{-1}$. It results from the high magnetic voltage (in A) between the left and the right side, linked with a high permeability $\mu_{TD} = 2500$. The corresponding relatively high induction $B_{TD} = 390$ mT determines a significant rotational magnetization (RM) in this sub-region (see Section V), as a source of high local losses.

(ii) A second portion proceeds along the left path of the middle limb, supported by high instantaneous permeability values in RD ($\mu_{RD} \approx 30000$). As a result, even more than 40% of the inner flux runs round the left window in the course of circulating magnetization (CM). The latter tends to increase local flux distortions, thus finally also increasing losses and harmonics of magnetostriction.

Summarizing, the given time instant is significant for the intensity of RM in the end region of the middle limb. Specific material parameter variations indicate that this intensity is strongly depending on the non-linear function $\mu_{TD}(B_{TD})$. As a kind of avalanche effect, increased slope yields strongly increased RM in connection with strongly reduced CM.

Fig. 5b shows results for the time instant when the magnetic flux in the outer T-limb is zero. A flux with mean induction $B = 1.47$ T comes from the R-limb and is taken over by the middle S-limb. Let us start our discussion in the R-limb and the following left yoke part. Here, the flux is distributed almost evenly over the inner and peripheral path, due to the already mentioned mechanisms. However, some overload of the window side results from shorter path length. Partly, the latter is compensated by the non-linearity of both functions $\mu_{RD}(B_{RD})$ and $R_{DD}(B_{DD})$.

The anisotropy and the impact of the T-joint overlap reluctances cause a complex flux distribution in the T-joint, as described also in [15], however for lower special resolution in
yokes. For the uptake of flux into the S-limb, we can distinguish three portions:

(A) A first portion arises along the inner path. It flows into the S-limb all along the RD in direct ways, with some uptake also from the periphery.

(B) This portion proceeds its way in the periphery, however losing continuously intensity. In the here given case of high anisotropy, it proceeds until the middle of the T-limb, thus going over to the upper core half in the course of circular magnetization (CM). With an intensity of $B = 0.14$ T, this case of CM is much weaker than CM in the S-limb (Fig.5a). The peripheral flux is compensated by anti-parallel inner one round the window.

(C) A third portion enters directly into the right S-limb path in the course of rotational magnetization (RM), being enhanced by the non-linear behaviour of $\mu_{TD}$. The latter increases up to 4000 corresponding to an absolute maximum of transverse induction $B_{TD} = 727$ mT. (Let us stress here, that such promille-data is the result of computing and does not mean a correspondingly high precision of modelling which in fact is a rough one.)

In the right yoke region, transverse flux is continued (and thus also RM). But due to the non-linearity of $\mu_{TD}$, its intensity sinks rapidly from from 329 mT next to the T-joint down to 17 mT in the central part of the outer T-limb. Since the total flux is zero, any flux along the peripheral path is linked with anti-parallel flux in the inner path. In the here assumed case of high anisotropy, a residual flux density of $B_{RD} = 0.14$ T reaches the center of the T-limb and continues its way round the window as circulating magnetization (CM). As already mentioned, anti-parallel instantaneous fluxes lead to flux distortions which themselves lead to increases of local losses and of harmonics of magnetostriction. Systematic variation of material parameters – in particular of the function $\mu_{TD}(B_{TD})$ - shows that the local intensities of all three RM, CM and anti-parallel fluxes depend on the materials anisotropy in extreme ways.

Calculations were also performed for lower magnetization. In contrast to $B_{NOM} = 1.7$ T, the evaluations for 1.3 T (not illustrated in the current paper) show that in the R-limb and also in the right yoke half, the flux is much more unevenly distributed with induction variations of more than 10 %. A main reason is that the compensating effect of the overlap reluctances to the shorter inner path length becomes much weaker due to the strong non-linearity of the function $R_{DD}(B_{DD})$ (see Fig.2c). The flux portion (A) becomes dominant, portion (B) becomes weaker, CM even disappearing (except for extremely high anisotropy), and portion (C) is reduced in distinct ways.

V. REGIONAL ROTATIONAL MAGNETIZATION PATTERNS

In the above, results of modelling have been discussed for four selected instants of time. Local distributions of induction were expressed by the two components $B_{RD}$ and $B_{TD}$. Now we will discuss time patterns of the induction vector

$$\mathbf{B}(t) = B_{RD}(t) \mathbf{e}_{RD} + B_{TD}(t) \mathbf{e}_{TD} \quad (3)$$
where $e_{RD}$ and $e_{TD}$ are the corresponding unit vectors in RD and TD. As it is well known - and also confirmed by the above described instantaneous results - the high anisotropy of the modern core materials yields the tendency that $B$ is directed in RD in most core regions. However, in T-joints and central yoke regions, $B$ leaves the RD in the course of rotational magnetization (RM). The intensity of RM is generally expressed by the axis ratio

$$a = \frac{\dot{B}_{TD}}{\dot{B}_{RD}} \approx \frac{\dot{B}_{TD,MAX}}{B_{NOM}} \quad (4)$$

with $\dot{B}_{RD}$ and $\dot{B}_{TD}$ the peak induction values in RD and TD, respectively.

As it is well known, RM has a great impact on the core performance. It may cause strong increases of losses and of magnetostriction (MS). However, these local increases depend not only on the ratio $a$, but also on the shape of the local magnetization pattern $B(t)$ and on its dynamics, i.e. on the instantaneous angular velocity $\omega(t)$ of $B$ [2,21]. During the cycle of magnetization, the latter shows strong variations. This means that effective modelling needs high resolution of time. In the present work, this was considered by 180 procedures of modelling for the halve period of magnetization.

We find almost mere alternating magnetization in the outer limb, and in good approximation also in the corner and the center of the middle limb (compare the experimental results of Fig.6b). On the other hand, the S-limb end and - in particular - the T-joint exhibit pronounced rotational magnetization.

The reconstruction of a magnetization patterns $B(t)$ is complicated by the fact that TD-elements are restricted to lamination centers, RD-elements to path centers. As an effective compromise, for an individual local pattern, the central induction component $B_{TD}$ was combined with a neighboring component $B_{RD}$. This procedure includes a spatial offset as a source of systematic error. However, specific variation tests indicated little impact on results, considering the a priori low spacial resolution of the model. Its results show principal agreement with experimentally observed ones as given in Fig.6b (after [2]) for a model core of similar type of material that was investigated by means of search coils.

Fig.6a shows eight examples of modelled magnetization patterns $B(t)$ for the different core regions. Both calculations and measurements prove that the outer limbs (locations L1 and L2) exhibit mere alternating magnetization with very small axis ratios $a$ that can be neglected in practice. Slightly higher values arise in the middle limb (L7, L8) and in the corners (L3). The model indicates relatively high values of the order $a = 0.2$ in the yoke (L4) next to the T-joint. It means that RM is not restricted to the small area of T-joint, but extends to central parts of the yokes. This is confirmed by the results of measurements that show values of the order $a = 0.1$.

As expected from the instantaneous results of Fig.4b, the model yields strongest rotational magnetization close to the end of V-element (L5, L6) with ratios up to $a = 0.44$. It also predicts a considerably high value of 0.23 at the end of the middle limb (L7). Measured ratios were lower here. However, such
differences are to be expected since different materials and geometries are given, different overlap parameters, apart from many sources of error of the two involved procedures of modelling.

Fig. 6. Magnetization patterns $B(t)$ in different regions of a 3-limb transformer core package for $B_{\text{NOM}} = 1.7$ T. (a) MACC-calculations of the local magnetization patterns and the corresponding local values of axis ratio $a$ for GO-material C130-30. (b) Measurement with search coils for GO-material M5 (after [2]).
According to the above, the two kinds of modelling yield same orders of regional axis ratios \( a \). In a rather unexpected way, they also yield comparable shape of the rotational magnetization patterns. In literature, most investigations of RM are restricted to elliptic magnetization patterns (e.g. [22,23]). A possible reason is that they are the only patterns that can be defined in simple ways, thus allowing exact comparisons between different types of materials. However, both the numerical and the experimental results of Fig.5 indicate that elliptical patterns do not arise in practice. Rather we find rhombic (or lancet-like) patterns [20,24-26].

One reason for rhombic patterns can be seen in the minimum permeability for the already mentioned hard directions of material. However, the numerical modelling indicates that also further mechanisms are involved. In the course of parameter variations, elliptical patterns according to Fig.7 were attained if the non-linear behavior of the material and the impact of the overlap reluctances was neglected. The permeabilities were set constant to \( \mu_{RD} = 30000 \) and \( \mu_{TD} = 2500 \), the overlap reluctances were set to \( R_{DD} = 0 \). This observation indicates that the practical patterns are affected by a large amount of impact factors.

![Fig.7. Numerically calculated rotational magnetization pattern, for the location directly under the V-element of the T-joint region for the case that the non-linearities of \( \mu_{RD} \) and \( \mu_{TD} \) as well as the impact of the overlap reluctances are neglected. The latter results in an elliptic pattern.](image)

Results of specific studies on phenomena of RM showed that its practical impact is not restricted to the axis ratio and the shape of pattern, but involves also the dynamics of rotation [2,20,21]. Fig.8a illustrates a numerically attained pattern for the location next to the V-element in detail. It indicates that the magnetization vector \( B \) rotates with a very specific angular velocity \( \omega(t) \). The latter tends to be very low in the vicinity of the RD. On the other hand, it increases gradually if the vector \( B \) rotates out of RD. The maximum \( \omega_{\text{MAX}} \) is reached when \( B \) passes through the TD. This maximum value determines the dynamics of \( B(t) \). It proves to depend strongly on the axis ratio, but obviously also on many other parameters. Thus it is rather surprising that the numerical result shows a high degree of agreement with an experimentally detected one as being shown in Fig.8b. In both cases (with \( f = 50 \) Hz), the maximum arises close to the 5\(^{th}\) millisecond, and its intensity is close to 100° per
millisecond.

Fig. 8. B-patterns with depiction of 20 instantaneous vector positions within the period of 20 ms for the location close to the V-element of the overlap region of the T-joint. At the right side, the corresponding angular velocity as a function of time for the half of the magnetization period. (a) Numerically calculated results for grain oriented material C130-30. (b) Measured results for a model core of grain oriented material 30M5 (after [2]; compare Fig. 6b). Notice: The four images are based on 180 instantaneous processes of modelling. However, marking of equidistant instants is restricted to ten, with time intervals of 1 ms (for 50 Hz).

With respect to the practical relevance of the above, it should be reminded that the losses are strongly affected by the shape and the angular velocity of the magnetization pattern. They sink for rhombic shape due to lower hysteresis losses, while they rise with \( \omega_{\text{MAX}} \) due to more pronounced eddy currents. The corresponding effects on the intensity of magnetostriction tend to be negligible [2]. However, increased dynamics tend to enhance the higher harmonics that are relevant for the generation of audible noise.
VI. DISCUSSION AND CONCLUSIONS

MACC is a novel methodology for numerical modelling of magnetic circuits like transformer cores. The performance of a core package can be effective modelled by as little as 50 flux elements. They are sufficient to describe the impact of most significant geometric parameters and material parameters. As a substantial advantage, the material can be described in multi-directionally non-linear ways. Due to the low number of elements, the computing time for the estimation of an instantaneous induction distribution is as short as about 1 s.

The present study demonstrates that the short computing time offers effective possibilities for quasi-dynamic modelling (i.e., so far, neglecting hysteresis). Sufficient time resolution is attained by a series of 180 snapshots of instantaneous induction distributions. Even a single snapshot may offer valuable conclusions on the impact of individual parameters. This is demonstrated in Sections III and IV by detailed descriptions of four individual snapshots. Single parameters of material or geometry can be varied in simplest ways, with minimum expenditure of time.

As demonstrated in Section V, the total of snapshots can be used to predict – or reproduce – time patterns $B(t)$ of local induction vectors. In a surprising way, they reflect detailed features of time patterns as determined on model cores in experimental ways. The advantage of MACC is that single parameters can be modified in a simple way. For example, it would be easy to estimate a function $a(b,h)$, i.e. the axis ratio $a$ of rotational magnetization in a given location as a function of window width $b$ and window height $h$). On the other hand, it would involve high expenditure to perform corresponding modifications of the geometry of a series of model cores. However, real models cannot be replaced by MACC since the results of the latter reveal that smallest changes of material parameters may show strong consequences, e.g. on the extent of rotational magnetization or circulating magnetization. That is, reliable numerical modelling would need precise data on permeability functions that are not available at present time.

As indicated by current works, the new methodology can be applied easily also for 3-dimensional modelling – e.g. of several packages in interaction. Also impact factors like DC-bias can be implemented. As a final conclusion, the methods spatial resolution is strongly restricted in order to keep computing times to acceptable levels and to allow for simple parameter variations. On the other hand, the method offers effective quasi-dynamic estimations of induction distributions with respect to their functional nature.

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REFERENCES
